unexpected. If the formula for a double diaphragm, one from each side in an unstepped waveguide, is examined, it is readily seen that the separate capacitance terms do not simply add. There is a mutual coupling between them, as would indeed be expected, accompanying the change of aperture field that one diaphragm induces on the other. Apparently, this rather general feature is absent in the two-to-one waveguide step with a junction diaphragm. Although the field is distorted by the diaphragm, the net excess charge due to the step is merely redistributed, a rather unexpected outcome. The formula has been checked by colleagues who find it correct, but who have no physical explanation for the finding. It is not known if it is a freak result coincidental on the two-to-one step ratio. Since, currently, the singular integral equation technique can only handle this case the effect of altering the step ratio on the diaphragm interaction is not, at the present time, resolvable.

Acknowledgment

The authors would like to thank, in particular, J. Tippet for the detailed check made on the derivation of the formula for the aperture field and junction susceptance.

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The Susceptance of an Annular Metallic Strip in a Circular Waveguide with Incident TE_{01} Mode

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***Abstract*—The principal aims of this paper are twofold: 1) to solve the problem of the scattering of a thin, perfectly conducting annular strip suspended in a multimodal circular waveguide in which any number of TE_{0n} modes can propagate, and with the aid of this result, 2) to give the susceptance of the thin annular strip in monomodal circular guide with an incident TE_{01} mode. These are treated with a variational approach.**

Applying the appropriate Green's functions to the continuity equations for the transverse electric field yields a variational expansion for the scattering matrix elements. This is treated with a Rayleigh–Ritz procedure and matrix methods.

Curves of normalized susceptance as a function of the free-space wavelength and the size of the annular metallic strip are shown. These results are in good agreement with experimental data.

Tables of the scattering coefficients for a typical wavelength versus strip size are also included.

I. Introduction

IN RECENT YEARS, an experimental millimeter-wave telecommunication system has been constructed in our country [1]. The circular waveguide capable of propagating the dominant circular-electric mode is ideally suited as a low-loss transmission line in the millimeter-

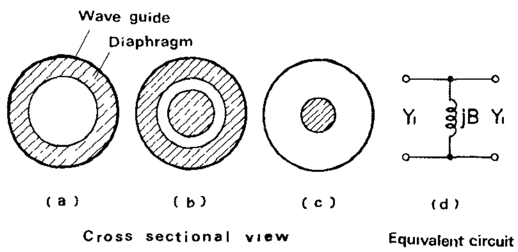

Fig. 1. Metallic plate diaphragms of zero thickness and their equivalent circuit (TE_{01} mode in circular waveguide).

wave region. Inductive metal irises of zero thickness in such circular waveguides (see Fig. 1) have been investigated and documented [2]–[4]. In these reports, the finding of the susceptance when the TE_{01} mode was incident was the main goal.

Problems of susceptance for discontinuities in waveguides have been widely studied during the past decade. Except for a few special discontinuities, exact solutions are not available and approximate methods must be used. Of the approximate techniques, the variational and integral-equation methods are applicable to a wide range of problems, and produce sufficiently accurate results for most purposes. The former method is described by Collin

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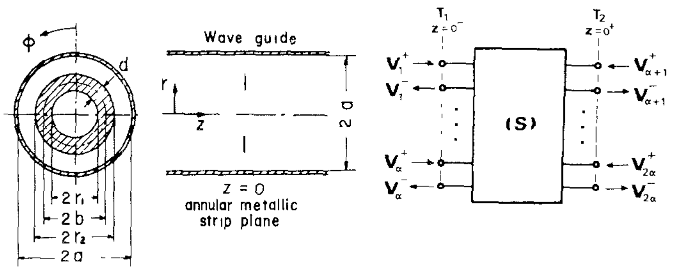

(a) (b) (c)

Fig. 2. Annular metallic strip of zero thickness and its equivalent network (TE_{0n}, $n=1,\ldots,\alpha$ modes in waveguide). (a) Cross-sectional view. (b) Side view. (c) Equivalent network.

[5], while the work of Lewin [6] is well known in connection with the latter.

Before the advent of high-speed computers, it was advantageous to expend considerable effort in manipulating solutions analytically into a form which minimized the subsequent computational effort. It is now often more convenient to use methods which are analytically simple, but require great amounts of computation. Furthermore, many problems of practical interest can be solved only by the use of such straightforward methods.

This paper is concerned with the determination using computerized methods of the susceptance and the scattering matrix of an annular metallic strip (see Fig. 2) to incident TE_{0n} modes. The annular strip placed in the circular waveguide in which the TE_{01} mode can propagate, but in which the higher circular electric modes cannot, acts as a shunt inductive susceptance, as shown in Fig. 1(d). If the higher TE_{0n} $(n=1,2,\cdots,\alpha)$ modes can also be assumed to propagate, on the other hand, the annular strip may be represented by a multiports equivalent network as shown in Fig. 2(c).

An annular strip might be useful, for example, in building a bandpass filter. Two annular metallic strips placed in the circular waveguide an integral multiple of half-wave lengths apart behave as a cavity resonator for the incident TE_{01} mode. These cavity resonators with a quarter wave length separation constitute a bandpass filter for the TE_{01} mode in the circular waveguide. A bandpass filter of this type has been shown by the authors [7].

Recently, a very ingenious technique for obtaining an equivalent network for this discontinuity in a rectangular waveguide with incident TE_{n0} mode has been presented by Rozzi *et al.* [8]. It is possible to obtain an equivalent wide-band network with frequency-independent elements. It is desirable to apply this technique to the present discontinuity in the circular waveguide, but straightforward application cannot be adopted due to the difficulties in analytical procedures for the circular coordinate system. Therefore, instead of that technique, we use a numerical method, requiring large amounts of computation, but analytically simple.

In the first part of this paper, the problem of the scattering coefficient is investigated for a thin, perfectly conducting annular strip suspended in a multimodal waveguide in which TE_{0n} $(n=1,2,\cdots)$ modes can propagate. Applying the appropriate Green's function to the continuity equations for the transverse electric field yields a variational expansion for the scattering matrix elements. This expansion is then treated with a Rayleigh–Ritz procedure and matrix methods.

In the second part of this paper, the susceptance of this annular strip in a monomodal circular guide with an incident TE_{01} mode is developed. The normalized susceptance B/Y_1 is obtained using the well-known relation $jB/Y_1=-2S_{11}/(1+S_{11})$ for the scattering matrix S.

II. Variational Expression of the Scattering Coefficient

The annular metallic strip shown in Fig. 2 is assumed to possess infinite conductivity and negligible thickness. The solution to this problem will be formulated in terms of a variational expression involving the current on the annular strip located at $z=0$.

Let a TE_{0j} mode be the incident mode. The incident electric field has only a ϕ component. Since the annular strip is uniform in the ϕ direction, the only higher modes excited by the annular strip are the TE_{0n} modes. The incident field excites a current distribution $I_j(r)$ on the annular strip. This current flows in the ϕ direction with a constant amplitude, since the incident field has no variation with respect to ϕ. The field excited by the annular strip in the waveguide may be evaluated in terms of currents on the annular strip using Green's function for the TE_{0n} modes.

With strip and r' denoting the surface and radius, respectively, of an annular strip, the excited field E_s is given by

$$E_s(r,z)=\int_{\text{strip}} G(r,z;r')I_j(r')\,ds' \quad [5]. \tag{1}$$

$I_j(r')$ is the current distribution excited by the incident TE_{0j} mode having unit amplitude. Therefore, $ds'=2\pi r'dr'$ $(r_1\leqslant r'\leqslant r_2)$, and

$$G(r,z;r')=-\frac{j\omega\mu}{2\pi a^2}\sum_{n=1}^{\infty}\frac{J_1(u_nr)}{\Gamma_nJ_0^2(\chi_n)}J_1(u_nr')e^{-\Gamma_n|z|} \tag{2}$$

[5], where $u_n=\chi_n/a$, $\Gamma_n=(u_n^2-k^2)^{1/2}$, ω is the angular frequency, μ is the magnetic permeability, a is the radius of the circular waveguide, $J_n(x)$ is the Bessel function of the first kind for order n, χ_n is the nth root of $J_1(x)=0$, $k=\omega(\mu\epsilon)^{1/2}=2\pi/\lambda$ is the free-space wavenumber, and ϵ is the electric permitivity.

It is evident, from (1) and (2), that the discontinuity excites an infinite number of modes. However, if the waveguide is standard or moderately oversize, only a limited number of modes appear at a point on the guide sufficiently removed from the strip.

Now consider the case in which the guide cross section is arranged so that only α modes propagate, the remaining modes being damped exponentially. In this case, a far-field equivalent circuit of the discontinuity is as illustrated in Fig. 2(c), where terminals T_1 and T_2 are placed at $z=0^-$, and 0^+, respectively. V_n^+ $(n=1,\cdots,2\alpha)$ and V_n^- $(n=1,\cdots,2\alpha)$ are the amplitudes of the electric field in the incident and reflected wave components, respectively, of the TE_{0n} mode field at terminals.

The transverse electric fields at the terminal plane ($z=0$) are described by

$$E_{\phi j}(r)=(V_j^+ + V_j^-)\phi_j(r) \tag{3}$$

where $\phi_j(r)$ are the mode functions in the transverse form of the TE_{0j} mode. The normalization of the mode functions is such that the total inward power flow at the terminal is given by $1/2(|V_j^+|^2-|V_j^-|^2)$; corresponding to the choice of unity as the characteristic impedance of the jth line [9]. The normalized mode function is described by

$$\phi_j(r)=\left(\frac{j\omega\mu}{\pi\Gamma_j}\right)^{1/2}\frac{1}{aJ_0(\chi_j)}J_1(u_j r). \tag{4}$$

From (1), (2), and (4), the reflected TE_{0i} mode is seen to be

$$E_{si}(r,z)=-\frac{1}{2}\phi_i(r)\int_{\text{strip}}\phi_i(r')I_j(r')\,ds'\cdot e^{-\Gamma_i|z|}. \tag{5}$$

Setting $z=0$ and $E_{si}(r,0)=R_{ij}\phi_i(r)$, we get

$$R_{ij}=-\frac{1}{2}\int_{\text{strip}}\phi_i(r')I_j(r')\,ds' \tag{6}$$

where R_{ij} is the reflection coefficient of the TE_{0i} mode for an incident TE_{0j} mode.

On the other hand, the following continuous equations for the conductive surface of the strip ($z=0$) are obtained for an incident TE_{0j} mode:

$$\phi_j(r)+\sum_{n=1}^{\infty}R_{nj}\phi_n(r)=\sum_{n=1}^{\infty}T_{nj}\phi_n(r)=0 \tag{7}$$

$$Y_j\phi_j(r)-\sum_{n=1}^{\infty}Y_nR_{nj}\phi_n(r)-\sum_{n=1}^{\infty}Y_nT_{nj}\phi_n(r)=I_j(r) \tag{8}$$

where $Y_n=\Gamma_n/j\omega\mu$ and T_{nj} is the transmission coefficient of the TE_{0n} mode for an incident TE_{0j} mode. Multiplying (8) by $\phi_k(r)$, and integrating over the cross section of the waveguide, we get

$$\int_{\text{guide}}\phi_k(r)I_j(r)\,ds=Y_j\int_{\text{guide}}\phi_j(r)\phi_k(r)\,ds - \sum_{n=1}^{\infty}Y_n(R_{nj}+T_{nj})\int_{\text{guide}}\phi_n(r)\phi_k(r)\,ds. \tag{9}$$

Using the orthogonal properties of the Bessel function, we have

$$\int_{\text{guide}}\phi_n(r)\phi_k(r)\,ds=\frac{\delta_{nk}}{Y_k} \tag{10}$$

where δ_{nk} is the Kronecker's delta. Combining (10) with (9), we get

$$\int_{\text{guide}}\phi_k(r)I_j(r)\,ds=Y_j\frac{\delta_{jk}}{Y_k}-\frac{Y_k}{Y_k}(R_{kj}+T_{kj}) \tag{11a}$$

hence,

$$(R_{kj}+T_{kj})=\frac{Y_j}{Y_k}\delta_{jk}-\int_{\text{guide}}\phi_k(r)I_j(r)\,ds. \tag{11b}$$

From (7) we have

$$\phi_j(r')+\sum_{n=1}^{\infty}((R_{nj}+T_{nj})\phi_n(r'))=0. \tag{12}$$

Substituting (11) into (12), we get

$$\phi_j(r')+\sum_{n=1}^{\infty}\left(\left(\frac{Y_j}{Y_n}\delta_{jn}-\int_{\text{guide}}\phi_n(r)I_j(r)\,ds\right)\phi_n(r')\right)=0. \tag{13}$$

The range of integration can be changed from guide to strip, because $I_j(r)=0$ at the point of aperture. Applying a property of Kronecker's delta,

$$\sum_{n=1}^{\infty}\left(\frac{Y_j}{Y_n}\delta_{jn}\phi_n(r')\right)=\phi_j(r') \tag{14}$$

(13) becomes

$$\phi_j(r')=\frac{1}{2}\sum_{n=1}^{\infty}\int_{\text{strip}}\phi_n(r)I_j(r)\,ds\cdot\phi_n(r'). \tag{15}$$

Multiplying by $I_i(r')$, and integrating over the annular strip, we obtain

$$\int_{\text{strip}}\phi_j(r')I_i(r')\,ds'=\frac{1}{2}\sum_{n=1}^{\infty}\iint_{\text{strip}}\phi_n(r)I_j(r)\cdot\phi_n(r')I_i(r')\,ds\,ds'. \tag{16}$$

A variational expression for R_{ij} may be obtained from (6) multiplying the right-hand side by

$$\int_{\text{strip}}\phi_j(r')I_i(r')\,ds'\Big/\int_{\text{strip}}\phi_i(r')I_j(r')\,ds'=1$$

and substituting (16) in as the denominator. We then have

$$R_{ij}=-\frac{\int_{\text{strip}}\phi_i(r')I_j(r')\,ds'\cdot\int_{\text{strip}}\phi_j(r')I_i(r')\,ds'}{\sum_{n=1}^{N}\iint_{\text{strip}}\phi_n(r)I_j(r)\phi_n(r')I_i(r')\,ds\,ds'} \tag{17}$$

where N is chosen for the truncation of the infinite expansion (16) to obtain suitably accurate numerical results.

The facts of the infinitesimal thinness of the annular strip, the symmetry about the plane $z=0$, and the reciprocity of the structure imply [10] that the scattering matrix has the form

$$\begin{bmatrix} V_1^- \\ \cdot \\ V_\alpha^- \\ V_{1+\alpha}^- \\ \cdot \\ V_{2\alpha}^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{1\alpha} & 1+S_{11} & S_{1\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ S_{1\alpha} & S_{\alpha\alpha} & S_{1\alpha} & 1+S_{\alpha\alpha} \\ 1+S_{11} & S_{1\alpha} & S_{11} & S_{1\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ S_{1\alpha} & 1+S_{\alpha\alpha} & S_{1\alpha} & S_{\alpha\alpha} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ \cdot \\ V_\alpha^+ \\ V_{1+\alpha}^+ \\ \cdot \\ V_{2\alpha}^+ \end{bmatrix}. \quad (18)$$

The problem is thus reduced to finding the element S_{ij} $(i=1,\cdots,\alpha,\ j=1,\cdots,\alpha)$ of the symmetric $\alpha\times\alpha$ matrix. The element S_{ij} is given by

$$S_{ij}=R_{ij} \quad (17')$$

because each line on which the TE_{0n} $(n=1,\cdots,\alpha)$ mode propagate has unit characteristic impedance.

Applying matrix algebra to the resultant scattering matrix, we also obtain the impedance and admittance matrices for the annular strip [4, p. 87].

III. Matrix Equation of the Scattering Coefficient for Numerical Calculation

To obtain an expression from (17), from which numerical values for the element S_{ij} of the scattering matrix may be calculated, a trial function $I_j(r)$ may be assumed.

For this discontinuity in a rectangular guide, a very ingenious technique for obtaining an appropriate trial function has been presented by Schwinger [6], [11], [12]. The appropriate set of basis functions, which lead to a trial function, were found by means of conformal mapping techniques, the quasi-static limit of the kernel summed in closed form, and the convergence guaranteed with the trial function of low order. It is desired to apply this technique to the present discontinuity in the circular guide, but straightforward application cannot be adopted due to the difficulties of analytical procedures for circular systems. Instead of such basis functions, accordingly, we use relatively simple basis functions.

A convenient approximation for the present discontinuity, therefore, is to assume the linear relation

$$I_j(r)=\sum_{m=1}^{M} C_{jm}\phi_m(r) \quad (19)$$

which will lead to expressions easily integrated. The trial function (19) requires large amounts of computation, but the calculated results obtained by using this trial function are in good agreement with experimental results obtained by Sheingold [2, Table I] and the authors.

Insertion of the trial function (19) into the expression (17) for annular strip scattering allows a variational form from the element of the scattering to be expressed as

$$\left(\sum_{s=1}^{M} C_{js}W_{is}\right)\left(\sum_{r=1}^{M} C_{ir}W_{jr}\right)+S_{ij}\sum_{n=1}^{\infty}\left(\left(\sum_{s=1}^{M} C_{js}W_{ns}\right)\cdot\left(\sum_{r=1}^{M} C_{ir}W_{nr}\right)\right)=0 \quad (20)$$

where

$$W_{ns}=\int_{\text{strip}}\phi_n(r)\phi_s(r)\,ds. \quad (21)$$

Determining the unknown coefficients C_{ir} and C_{js} so as to make the scattering coefficient S_{ij} stationary [13], and using standard matrix techniques [5], we get the relation for the scattering coefficient

$$S_{ij}=-a_{11} \quad (22)$$

for the following inverse matrix A:

$$\begin{bmatrix} \dfrac{g_{11}}{W_{i1}W_{j1}} & \dfrac{g_{12}}{W_{i2}W_{j1}}-\dfrac{g_{11}}{W_{i1}W_{j1}} & \cdots & \dfrac{g_{1M}}{W_{iM}W_{j1}}-\dfrac{g_{11}}{W_{i1}W_{j1}} \\ H_{21} & H_{22}-H_{21} & \cdots & H_{2M}-H_{21} \\ \cdots & \cdots & \cdots & \cdots \\ H_{M1} & H_{M2}-H_{M1} & \cdots & H_{MM}-H_{M1} \end{bmatrix}^{-1} \quad (23)$$

$$g_{rs}=\sum_{n=1}^{\infty} W_{ns}W_{nr} \quad (24)$$

$$H_{rs}=\frac{g_{rs}}{W_{is}W_{jr}}-\frac{g_{1s}}{W_{is}W_{j1}}. \quad (25)$$

The integrals in (21) can be evaluated using these standard forms for Bessel functions [2]

$$\int_{r_1}^{r_2} rJ_1(u_n r)J_1(u_s r)\,ds = \frac{a^2\delta_1}{\chi_s^2-\chi_n^2}(\chi_s J_1(\chi_n\delta_1)J_0(\chi_s\delta_1) - \chi_n J_1(\chi_s\delta_1)J_0(\chi_n\delta_1)) - \frac{a^2\delta_1}{\chi_s^2-\chi_n^2}(\chi_s J_1(\chi_n\delta_2)J_0(\chi_s\delta_2) - \chi_n J_1(\chi_s\delta_2)J_0(\chi_n\delta_2)) \quad (26a)$$

$$\int_{r_1}^{r_2} r(J_1(u_n r))^2\,ds = \frac{a^2\delta_1^2}{2}(J_0(\chi_n\delta_1)J_2(\chi_n\delta_1)-J_1^2(\chi_n\delta_1)) - \frac{a^2\delta_2^2}{2}(J_0(\chi_n\delta_2)J_2(\chi_n\delta_2)-J_1(\chi_n\delta_2)) \quad (26b)$$

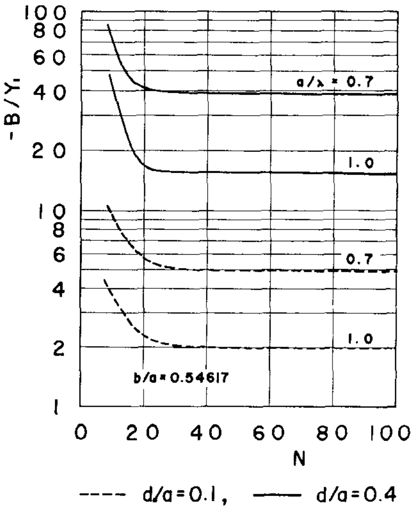

Fig. 3. Normalized susceptance of annular strip versus number of terms N. $M=15$, $b/a=0.54617$.

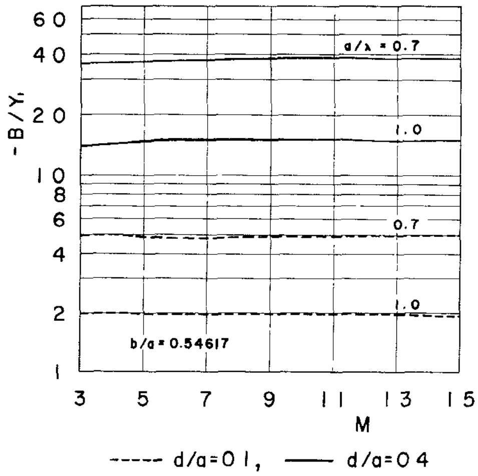

Fig. 4. Normalized susceptance of annular strip versus number of terms M of the series representing an annular strip current. $N=100$, $b/a=0.54617$.

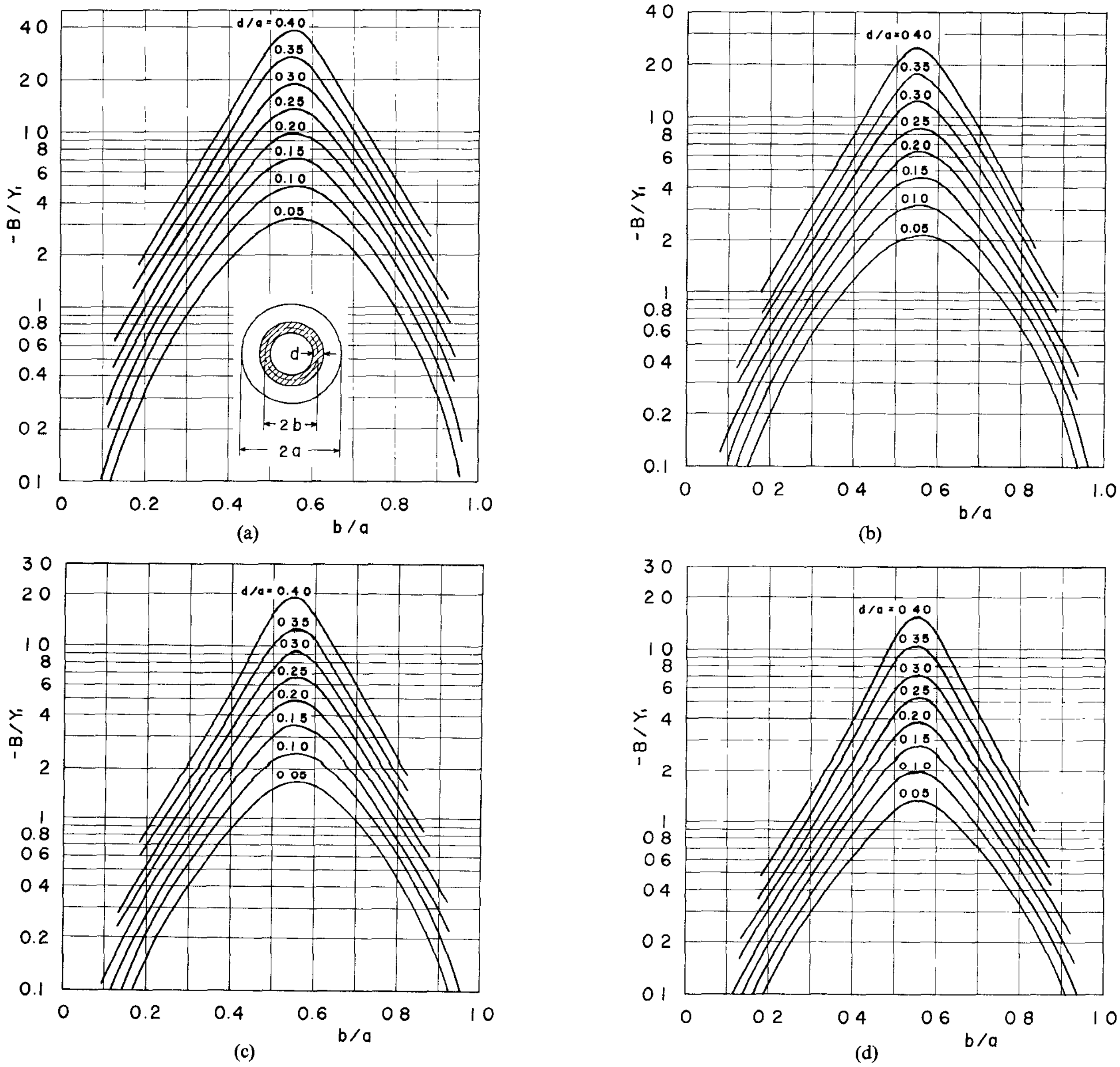

Fig. 5. Normalized values of the annular strip susceptance as a function of average radius, with its width and free-space wavelength as parameters. (a) $a/\lambda=0.7$. (b) $a/\lambda=0.8$. (c) $a/\lambda=0.9$. (d) $a/\lambda=1.0$.

where

$$\delta_1 = \frac{r_1}{a} \quad \delta_2 = \frac{r_2}{a}. \tag{27}$$

IV. Susceptance of the Annular Strip in a Circular Guide with Incident TE_{01} Mode

An annular metallic strip placed in the circular waveguide in which the TE_{01} mode can propagate but in which higher circular-electric modes cannot, acts as a shunt inductive susceptance. The purposes of this section is to calculate the susceptance of the annular strip in circular guide with incident TE_{01} mode.

The interrelation between a shunt susceptance jB/Y_1 across a transmission line with unit characteristic impedance and a scattering coefficient S_{11} is given by

$$j\frac{B}{Y_1} = -\frac{2S_{11}}{1+S_{11}} \tag{28}$$

where Y_1 is the characteristic admittance for a TE_{01} circular electric transmission line.

In order to show the effects of truncation of the series (17) and (19), the calculated susceptance of the annular strip is shown in Figs. 3 and 4 as a function of the numbers of the terms. With reference to these results, the numbers of terms N and M can be determined. We set $N=100$ and $M=7$ for numerical computations of the susceptance shown in subsequent figures.

Variation of the average radius b/a of an annular strip yields curves of susceptance shown in Fig. 5 with annular strip width d/a as parameter. It can be seen that maximum susceptance occurs at $b/a=0.54617$, where the incident electric field E_ϕ is maximum as was expected.

Fig. 6 shows curves of susceptance for the annular strip, of which the outside edge is in contact with inside wall of the waveguide ($r_2=a$), as a function of the inside radius r_1 of the strip for the frequency $a/\lambda=0.7$. Dots in Fig. 6 show the experimental values presented by Sheingold [2, Table 1]. The theoretical results are in excellent agreement with the experimental values. From this result, we know that the trial function (19) used in the computation is quite suitable for practical purposes [14].

Fig. 7 shows the variation of the scattering coefficients of the higher TE_{0n} modes as a function of the average radius of an annular strip with d/a as a parameter. The scattering of the higher mode is sharply decreased at specific average radii which depend on the value of n. From Fig. 7, we can determine a suitable average radius to restrain the generation of the TE_{02} mode. This radius is approximately equal to $b/a=\chi_1/\chi_2=0.54617$ which is obtained with

$$G_{02}(r,z;b)=0 \quad \text{or} \quad J_1(\chi_2 b/a)=0 \tag{29}$$

where G_{02} represents the second term of Green's function (2). This equation means that no field of the TE_{02} mode arises from a current along an infinitesimally thin annular strip. In the following, therefore, we carry out the calculations for an annular strip having such an average radius.

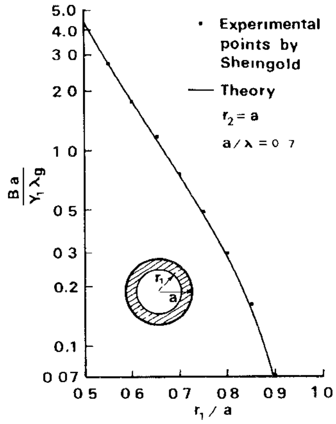

Fig. 6. Normalized susceptance of the obstacle of which outside radius is equal to the inside radius of the circular waveguide.

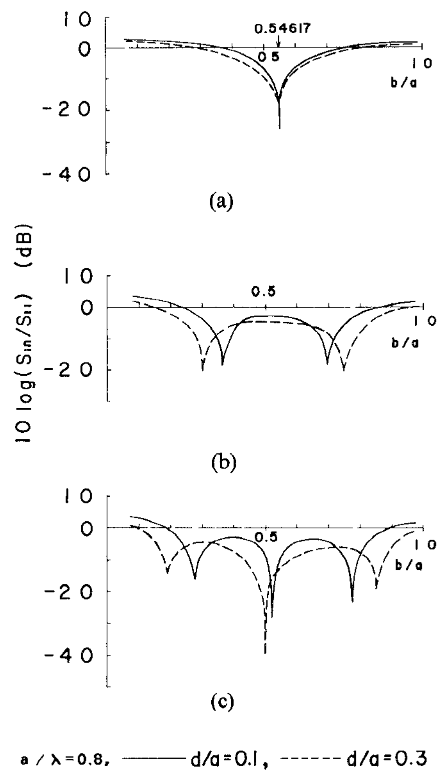

Fig. 7. Scattering coefficients of (a) TE_{02}, (b) TE_{03}, (c) TE_{04} modes for incident TE_{01} mode as a function of average radius of the annular strip, with its width as a parameter. (a) $n=2$. (b) $n=3$. (c) $n=4$.

The normalized susceptance of (28) are plotted in Fig. 8 as functions of d/a for values a/λ over the range $0.65 \leqslant a/\lambda \leqslant 1.10$. These theoretical results are in good agreement with experimental values.

Susceptances of the annular strips were measured by the tangent method [5, p. 215]. The inside diameter of the circular waveguide which we used for measurements is 51 mm. The experiments were made in the frequency range $8.8 \leqslant f(\mathrm{GH}) \leqslant 10.0$, such that in the waveguide all TE_{0n} modes except the TE_{01} mode were in a cutoff region. The annular metallic strips were made by etching from a copper plate of 0.07-mm thickness. The annular metallic strips were supported by a polyfoam column of which dielectric properties were quite close to those of air.

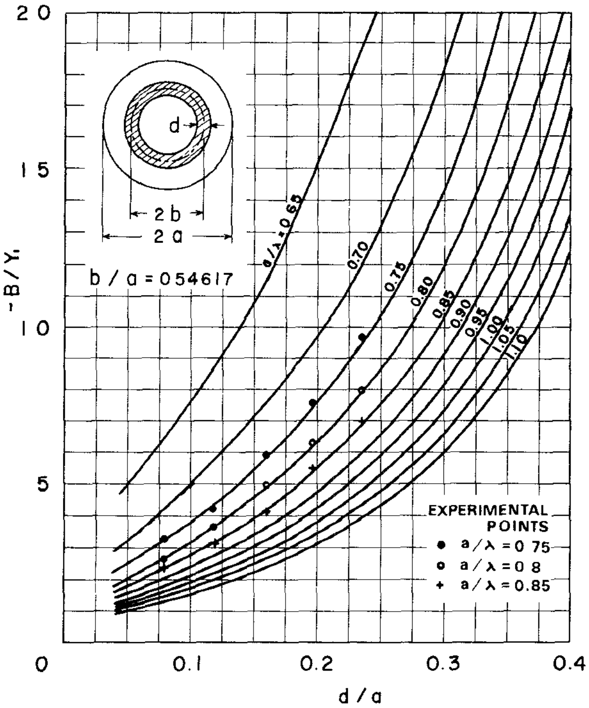

Fig. 8. Normalized values of the susceptance of the annular strip as a function of its width, with a/λ as a parameter.

TABLE I
COMPUTED SCATTERING MATRIX ELEMENTS FOR ANNULAR METALLIC STRIP

i \ j	1	2	3
1	-0.335+0.059i	0.006+0.024i	0.466-0.038i
2	0.006+0.024i	-0.026+0.148i	-0.018+0.016i
3	0.466-0.038i	-0.019+0.015i	-0.653+0.085i

$d/a = 0.2$, $b/a = 0.546$, $a/\lambda = 1.7$, $i = \sqrt{-1}$

TABLE II
COMPUTED SCATTERING COEFFICIENTS OF ANNULAR STRIPS AS A FUNCTION OF ITS WIDTH

d (mm)	S_{11}	S_{12}	S_{13}	S_{22}	S_{23}	S_{33}
1	-0.188+0.121i	0.000+0.000i	0.312-0.201i	0.000+0.007i	-0.001+0.001i	-0.519+0.335i
3	-0.269+0.076i	0.005+0.008i	0.422-0.112i	-0.002+0.054i	-0.008+0.007i	-0.664+0.179i
5	-0.337+0.061i	0.006+0.022i	0.424-0.069i	-0.021+0.145i	-0.018+0.015i	-0.656+0.091i
7	-0.408+0.087i	-0.003+0.048i	0.480+0.027i	-0.094+0.278i	-0.035+0.030i	-0.603+0.072i
9	-0.533+0.163i	-0.043+0.062i	0.446+0.133i	-0.255+0.427i	-0.077+0.039i	-0.570+0.144i
11	-0.734+0.214i	-0.086+0.019i	0.299+0.227i	-0.566+0.467i	-0.124-0.009i	-0.661+0.258i
13	-0.913+0.153i	-0.058-0.035i	0.113+0.194i	-0.824+0.353i	-0.088-0.075i	-0.848+0.253i
15	-0.982+0.068i	-0.017-0.033i	0.265+0.100i	-0.953+0.192i	-0.030-0.068i	-0.959+0.150i

$a = 25.5$ mm, $b = 13.9$ mm, $f = 20$ GH, ($b/a = 0.546$, $a/\lambda = 1.7$)

V. SCATTERING MATRIX OF THE ANNULAR STRIP IN A CIRCULAR GUIDE WITH INCIDENT TE_{0n} MODES

As an example of the numerical computation of (17), the scattering matrix of the annular strip is calculated for the frequency $a/\lambda = 1.7$ at which higher TE_{0n} $(n = 1,2,3)$ modes can propagate. Table I shows the calculated scattering matrix.

Certain general properties of the scattering coefficients may be deduced from general principles [9]. In the following, we will check whether these properties are satisfied or not by inspection of the numerical values of the scattering coefficients given in Table I.

A. The Reciprocity Relations

From Section II, the normalized mode functions have been defined by (4). This corresponds to choosing an equivalent characteristic impedances equal to unity. The scattering matrix, therefore, must be symmetrical [9], $S_{ij} = S_{ji}$.

By inspecting the values of the scattering coefficients given in Table I, we see immediately that the symmetrical condition is satisfied with only a very small amount of error.

B. The Unitary Relations

The structure under consideration is nondissipative, hence, the power-conservation condition must be satisfied. This condition can be reduced to the unitary condition of the scattering matrix by means of matrix algebra [9]

$$[S]_t [S]^* = [U]. \tag{30}$$

Inserting the values of elements of the scattering matrix given in Table I into the left-hand side of (30), we confirm that the values of elements in the resultant matrix deviate from the unit matrix within 6/1000. From this fact, it is known that the unitary condition is approximately satisfied.

The scattering coefficients are tabulated in Table II as a function of the strip width d with $a/\lambda = 1.7$ at which TE_{0n} ($n = 1,2,3$) modes propagate. From these results, we see that, first, the values of the scattering coefficients having subscripts $i = 2$ or $j = 2$ are very small for narrow strips, because the average radius of the annular strip was chosen as $b/a = 0.54617$ in connection with (29), and second, the scattering coefficient S_{11} which is relevant to a shunt susceptance increases from nearly 0.2 to 0.98 according to the increase in the strip's width d from 1 to 15 mm.

VI. Conclusion

The main results of the analyses and examples presented in this paper are as follows.

1) The variational expression of the scattering matrix for an annular metallic strip in a circular waveguide in which multimode (TE_{0n}, $n = 1, \ldots$) propagate have been given.

2) Using the matrix element S_{11} of the scattering matrix, the susceptance of the annular strip in a circular guide has been calculated for the TE_{01} mode propagation. A numerical calculation gives results which agree closely with the values obtained by experimental methods over a wide range of conditions.

3) Certain modes can be suppressed from the scattered waves by choosing the average radius of the annular strip. This could be useful in some application where a particular mode is harmful.

4) Numerical results of the scattering matrix were obtained. For simplicity, only the modes TE_{0n}, $n = 1,2,3$ have been assumed to propagate. It is easy to see, however, that higher TE_{0n}, $n = 4,5,\ldots$ modes can also be assumed to propagate without unduly increasing the computation time. Results derived here, therefore, should be easily applied to propagation in oversized circular waveguides such as those commonly used with millimeter wavelengths.

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